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REPORT

CEEFAX: tests on three possible choices of secondary code

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Summary

Coded data-signals for the CEEFAX data-broadcasting system are combined with the picture signal before transmission. The whole television signal is subject to noise and waveform distortion within the transmission path which can give rise to information errors when the data-signal is decoded in a CEEFAX receiver.

This Report describes a comparison of three binary codes suitable for the transmission of coded data on the television waveform. These three Non-Return-to-Zero (NRZ) code, BiPhase Level (BiφL) code and Delay Modulation (DM) code have been compared on theoretical and practical grounds for a practical data-broadcasting system. Ruggedness in the face of signal distortion and noise, possible data-rate within the given bandwidth and the complexity of decoding circuits in CEEFAX receivers have all been taken into account. It has been assumed in this Report that receivers for CEEFAX will incorporate synchronous detection (i.e. data will not suffer quadrature distortion), a.f.c. (i.e. tuning errors will be negligible) and that the overall group delay distortion will be small.

The results indicate that NRZ code offers the best choice for a coded-data-broadcasting system.

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CEEFAX: TESTS ON THREE POSSIBLE CHOICES OF SECONDARY CODE

N.E. Tanton, B.A.

Glossary of Symbols

A	Peak to peak signal amplitude
A	Numerical constant
B	Numerical constant
C	Numerical constant
$G(s)$	Voltage response in complex frequency domain
s	Complex angular frequency $s = \sigma + j\omega$
$S^*(\omega)$	Spectral density of raised-cosine data pulses
t	Time
τ	Time delay
T	Bit-cell clocking interval
$U(t)$	Unit voltage step
ω	Angular frequency, imaginary part of s
σ	Angular frequency, real part of s

1. Introduction

In a data transmission system, a data-code should be used which best suits the characteristics of the particular transmission channel. Whilst a specific data-code may be convenient as a 'primary code' for data-processing at either end of the transmission path, it is often necessary to re-code the data-signals into some 'secondary-code' before transmitting the data, in order that the best use may be made of the transmission channel.

In the early form of the BBC CEEFAX system, data (including error-protection) in eight-bit words was transmitted serially as Non-return-to-zero (NRZ)* data on lines 17, 18, 330 and 331 of the 625-line television waveform. In this case, the serially-read words required no further coding to produce NRZ-coded signals. At the same time, in the proposed IBA system, ORACLE, the primary-coded data signals were further coded into BiPhase Level code (BiϕL)* before being transmitted in the television waveform. As a result of field trials and the results described in this Report, NRZ code is being used for the unified data broadcasting system in the UK.

It is important that the data transmitted should be as resistant as is practicable to noise and all forms of interference, whilst retaining as high as possible a data bit-rate; in addition receivers should be as simple as possible. In this Report, NRZ code, BiϕL code and a third binary code, Delay Modulation (D.M.),* are considered as secondary codes and compared, with regard to noise resistance, in the light of their possible use in a data-broadcasting system such as CEEFAX or ORACLE.

It has been assumed in this Report that receivers for CEEFAX will incorporate synchronous detection (i.e. data will not suffer quadrature distortion) and a.f.c. (i.e. negligible tuning errors) and that the overall group-delay distortion in the signal path (including the transmitter and receiver) will be low.

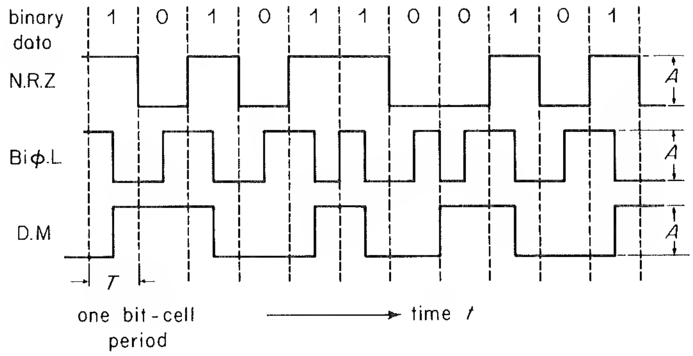

Fig. 1 - Waveforms for rectangular pulses of Non-Return-to-Zero (NRZ), BiPhase Level (BiϕL) and Delay Modulation (DM) coded binary data

2. Theoretical investigation: fundamental characteristics of NRZ, BiPhase Level and Delay Modulation codes

The three codes considered are all binary codes and are depicted in Fig. 1 in terms of the binary data encoded.

2.1. Basic code format

Conventionally, NRZ code comprises a two-level signal adopting a low level for a bit-cell period if the signal represents logic-low, and a high level for a bit-cell period if the signal represents logic-high. In Fig. 1, the signal excursion (A) represents the difference between the two signal levels. BiϕL code involves transitions at each bit-cell centre, as shown in Fig. 1, from the low to the high level if logic-low is to be represented, and from the high to the low level if the data-signal is logic-high. When successive message bits are alike, transitions also occur at bit-cell boundaries.

D.M. code involves transitions in either direction at bit-cell centres only if the signal represents logic-high; transitions also occur between successive logic-low bits at the bit-cell boundaries.[1] Thus D.M.-coded signals can be derived by passing BiϕL-coded signals through a modulo-2 counter.

2.2. Raised-cosine filtering

As rectangular pulses, the codes described have spectra extending, in principle, to infinity in the frequency domain; therefore, in order to limit the energy distribution of a sequence of data pulses, the signals must be filtered. 'Raised-cosine' filtering has the advantage that relatively

* See Section 2.1 and Fig. 1.

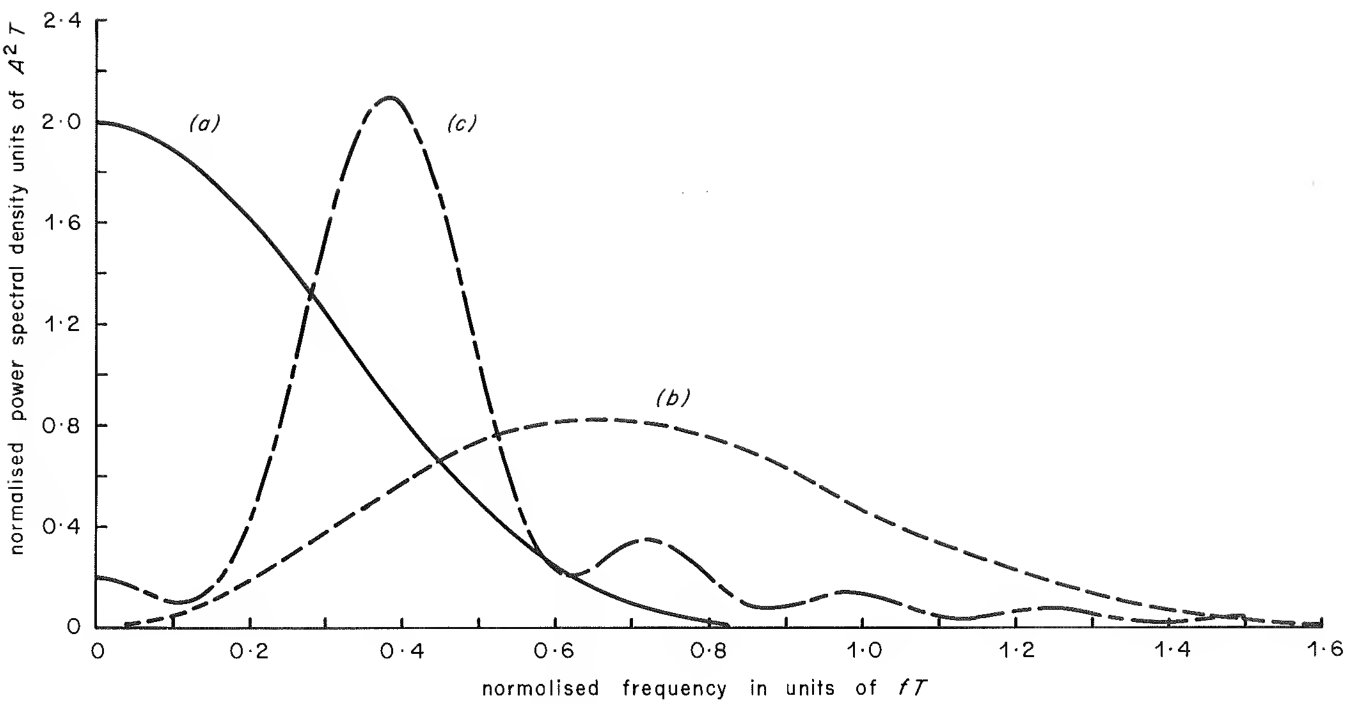

Fig. 2 - Power spectral densities for random data sequences

(a) ———— raised-cosine filtered NRZ (b) - - - - - - - raised-cosine filtered BiϕL (c) —— — —— raised-cosine filtered D.M.
A peak-to-peak signal amplitude f frequency T bit-cell period

little energy extends beyond the data-rate (this applies for the spectra of NRZ- and D.M.-coded data more so than for BiϕL data). Moreover a raised-cosine filter has a time-bounded impulse-response so the intersymbol interference* is less than with for example a sharp-cut low-pass filter with an impulse-response approximating to $(\sin x)/x$.

BiϕL and D.M. codes have a clocking interval of half a bit-cell period, whereas NRZ code has a clocking interval of one bit-cell period. As a result the nominal bandwidth of the raised-cosine filter used for a BiϕL code or D.M. code is twice that required for NRZ code assuming the same overall data rate (see Appendix B).

2.3. Power spectral densities for raised-cosine secondary codes

The calculated power spectral densities of raised-cosine data for the three coding methods are shown in Fig. 2 and described in Appendix A. The curves in Fig. 2 are normalised so as to correspond to a given data rate and assuming a random sequence of binary data-pulses.

2.4. Theoretical comparisons of the three codes

Several points arise from Figs. 1 and 2:

(a) At the receiving end of a data-transmission channel it is desirable that clock-synchronising information be obtainable from signal transitions in the data, as well as from any clock-synchronising signals specially included in the data stream. NRZ-coded signal may have extremely long intervals between data transitions. On the other hand, BiϕL-coded signals provide at least one transition per bit-cell, irrespective of the data content; similarly D.M. code gives a transition at least once every two bit-cells. Timing information may therefore be extracted from the data message when using either BiϕL or D.M. code more readily than when using NRZ code.

(b) It is necessary to sample both BiϕL and D.M.-coded signals twice per bit-cell so that they may be decoded unambiguously; it is therefore necessary to provide a train of sampling clock-pulses running at twice the bit-rate. For NRZ-coded signals, only one sample per bit-cell need be taken and, furthermore, no decoding is required to present the data as binary information.

(c) BiϕL has the property that the two halves of each bit-cell have opposite signal values. The signal may therefore be checked for single errors without adding extra protection bits, provided that the error rate is sufficiently low (about 10^{-2} or less) that double errors are very unlikely.

However, protection bits must be included if any attempt is to be made to correct errors thus detected.

(d) A random-sequence NRZ signal includes a strong d.c. component; indeed it is at zero-frequency that the power spectral-density for NRZ signals is at a maximum. BiϕL contains no d.c. component, whilst that for D.M. is an order of magnitude less than that for NRZ.

(e) At a given data bit-rate, NRZ-coded signals occupy considerably less bandwidth than BiϕL-coded signals, and

* Intersymbol interference is interference between adjacent data pulses.

somewhat less than that required for D.M.-coded signals. Indeed, with raised-cosine filtering, 97% of the power of an NRZ signal is contained below 0·6 of the message bit-rate, whilst 84% of the D.M. signal power and only 38% of the BiϕL signal power are included within this bandwidth. Using raised-cosine filtering, BiϕL coding requires at least twice the bandwidth for a given message bit-rate than NRZ code.

2.5. Codes as used in CEEFAX

Several comments on the comparisons of 2.4 are relevant when applying the codes to a real data-broadcasting system, such as CEEFAX.

(a) In the CEEFAX system, each character is described by an eight-bit data word, of which the eighth bit is an odd-parity bit, included at the end of the word. Because of this parity bit, there is always at least one signal transition every word, and so a transition (from which timing information could be retrieved) occurs at least twice every sixteen data bits. Provided that the circuit used in the receiver for extracting timing information has a sufficiently long time-constant, no difficulty is encountered in maintaining clock-pulse synchronism using CEEFAX data-words.

(b) The relatively strong d.c. component in an NRZ code complicates the problem of establishing a suitable slicing level (for decisions as to whether data is logic-high or logic-low) at the receiver. BiϕL has no d.c. component and its mean level provides a suitable slicing level. The problem with NRZ is greatly eased in practice because it is convenient to make one of the two logic levels the same as black-level in the television waveform. Data is then sliced halfway between the maximum data signal level and the minimum signal level.

(c) In order to establish clock-pulse synchronism for each data-line at the receiver, a 'clock-start' sequence is transmitted at the beginning of each line of data. The sequence is designed to provide sufficient regular signal transitions for a timing circuit (e.g. a 'flywheel' tuned circuit at the clock-rate frequency) to achieve synchronism with the data-signal by the time the first data-word is presented to the receiver for processing; this sequence gives rise to a strong spectral component at half the bit-rate. Furthermore, the parity bit in each eight-bit word gives rise to spectral components at harmonics of one eighth of the bit-rate. The spectrum of a random-sequence NRZ-coded signal, shown in Fig. 2, is therefore modified in practice, so as to have the same overall envelope as in Fig. 2, but with a line-spectrum of components at field-rate (due to the presentation of data once every field) and additional 'line-spectra' at harmonics of one-eighth bit-rate frequency.

2.6. Basis for the choice of secondary code

The various points considered when choosing the secondary code for the data-broadcasting system were as follows:—

1. the error-rates suffered by particular codes in the presence of various relevant forms of interference,

2. for a given error-rate, the highest data bit-rate possible within the given bandwidth (somewhat less than the luminance bandwidth of 5·5 MHz in System I television*); this determines the number of pages of information which can be transmitted in a given time interval,

3. the complexity of decoding circuits required at the receiver; in principle, the simpler the decoding circuit the cheaper and more reliable the receiver can be made. This conflicts to some extent with the possibility of an exchange between (1) and (2); for example, a fast code may have inferior resistance to noise, but with additional error-protection (and receiver complication) might be made sufficiently resistant to interference whilst still retaining some of its speed advantage.

Errors in data-transmitting systems are usually introduced by distortion and/or noise accompanying the received data-signal. If, when the signal is sampled (in order to decode the data), the distortions or the distortions-plus-noise cause the signal level, at the sampling instant, to lie on the wrong side of the slicing level, the data will be incorrectly decoded.

Bandwidth-limited data-signals suffer from intersymbol interference, the amount of distortion depending, for a given data-code, on the amount of bandwidth-limiting. The combination of signal-distortion and interference may make one code more susceptible to errors than another. Noise has been used in the work described in this Report to assess the suitability of a particular secondary code for data-broadcasting, bearing in mind the distortions and interference that the transmission channel can impose on a data-signal.

It has been assumed that receivers for CEEFAX would incorporate a.f.c. and synchronous detection. This would eliminate unnecessary bandwidth limiting and other distortions resulting from tuning errors and would also avoid quadrature distortion.

3. Experimental investigation

3.1. Bandwidth requirements

To determine the bandwidth requirements of the various codes, two approaches were possible. The bit-rate could be varied using a fixed, known bandwidth or the bandwidth could be varied using a given bit-rate; in practice, the latter method was found to be more convenient.

3.2. Method

Raised-cosine data signals were further bandwidth-limited using various low-pass filters having impulse responses approximating to $(\sin x)/x$. The resistance to noise of the coded signals thus processed was measured by de-

* Because of the possibility of using the same signals for Systems B and G.

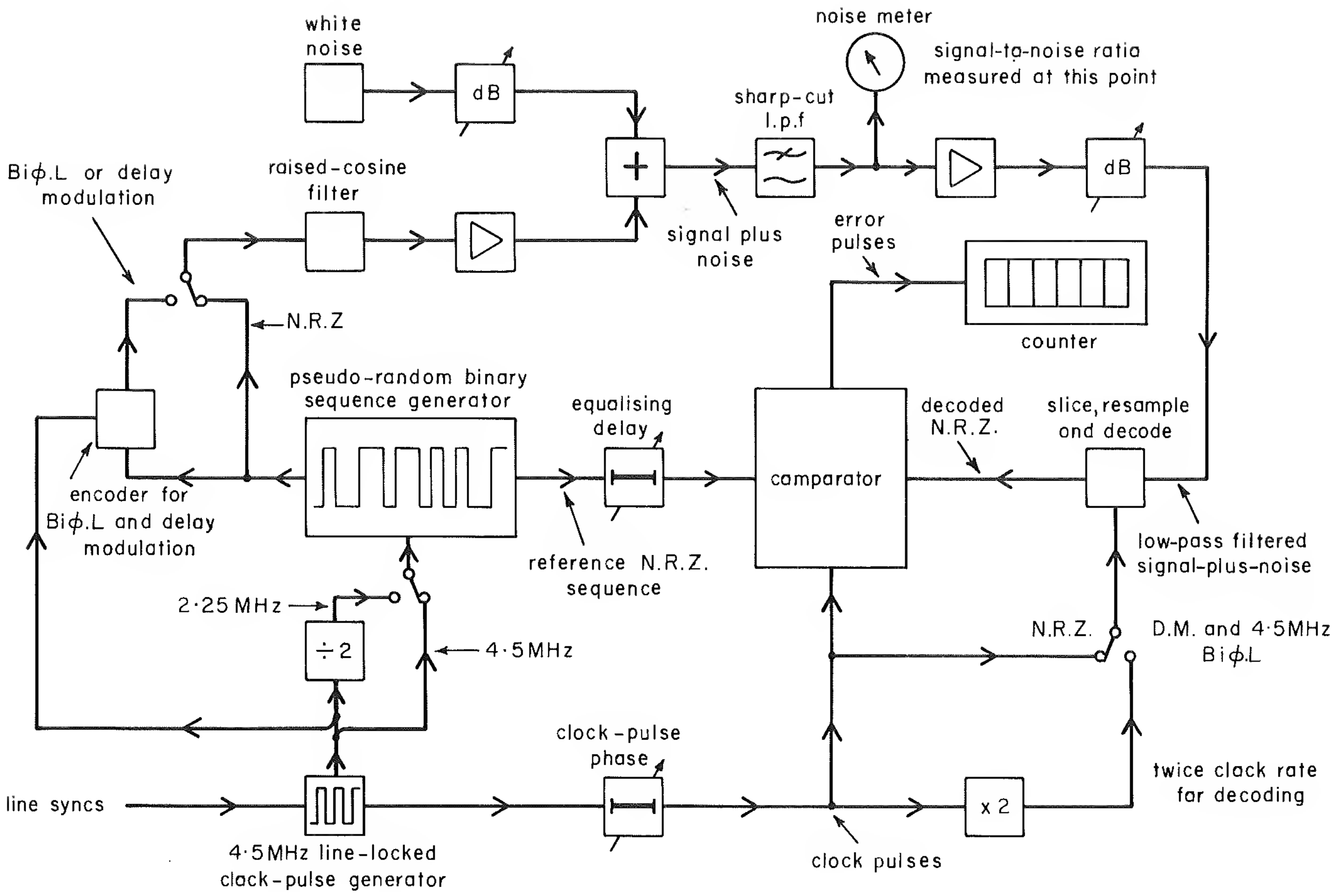

Fig. 3 - Experimental arrangement for comparing base-bandwidth limited coded data signal in the presence of white noise

coding the processed data-plus-noise and comparing the decoded data with the original data.

Referring to Fig. 3, a pseudo-random sequence of NRZ-coded data (of sequence-length 288 bits) was generated at either of two bit-rates, namely 2·25 Mbits/sec or 4·5 Mbits/sec, and was derived from a 4·5 MHz clock-pulse generator synchronised to a multiple of television line-frequency. The sequence was triggered by line-synchronising pulses and gated for the active line period of the television waveform (52 μsec). In this way, data-blocks at television-line repetition rate were formed.

This data-sequence was then, in the case of BiϕL coding and D.M. coding, further coded into the appropriate form (NRZ data required no further coding). The resultant signal (at a bit-rate of either 2·25 Mbits/sec or 4·5 Mbits/sec) was passed through a filter of appropriate design (see Section 2.2 and Appendix B), to emerge as a sequence with 'raised-cosine' edges. White noise was then added to the filtered data and the data and noise were low-pass filtered using a linear-phase low-pass filter;[2] several filters with different cut-off frequencies were used to investigate the bandwidth requirements of the coding methods as described earlier in Section 3.1. After level-adjustment to compensate for the attenuation of the low-pass filter, the signal was fed to a slicing, resampling and, where applicable, decoding circuit in order to provide data in a form suitable for comparing the pseudo-random sequence with an unimpaired version of itself.

It is in the slicing and resampling circuit of the receiver in any transmission channel that noise and other forms of interference can cause errors; the noise and/or interference may cause the signal to lie on the wrong side of the decision level, at the instant of resampling.

Therefore, in any data-broadcasting receiver, the data must be sliced to define logic-high and logic-low levels and resampled to exclude as far as possible the effects of pulse width modulation by noise. In this experiment, a black-level clamp in the slicing circuit was triggered by a suitably timed pulse to establish the logic-low voltage level at the beginning of each line of data signal. The slice level was then offset from black level to lie at the centre of the data signal swing.

For BiϕL- and D.M.-coded data signals the data-bits must be resampled, once in the first half and once in the second half of each bit-cell, in order to decode the bits unambiguously. The data was sampled in a D-type flip-flop clocked with either 4·5 MHz clock-pulses (for 4·5 Mbits/sec NRZ- or 2·25 Mbits/sec BiϕL-coded data) or with 9·0 MHz clock-pulses (for 4·5 Mbits/sec BiϕL- or D.M.-coded data). The instant of resampling was adjusted relative to signal transitions; for undistorted NRZ-coded data the optimum sampling instant is halfway along the bit-cell.

TABLE 1

NRZ Compared with BiφL

	NRZ (4·5 Mbits/sec)		BiφL (2·25 Mbits/sec)	
Bandwidth (−3 dB) MHz	Bandwidth / Bit-rate	Signal-to-noise ratio for 1 in 10^5 error rate	Bandwidth / Bit-rate	Signal-to-noise ratio for 1 in 10^5 error rate
5·5	1·22	19·5 dB	2·44	19·4 dB
4·5	1·00	19·8 dB	2·00	20·1 dB
2·8	0·62	19·1 dB	1·24	19·3 dB
2·4	0·53	19·4 dB	1·06	20·3 dB
1·6	0·36	see below	0·71	—

As has been described, clock-synchronising information in a data-broadcasting receiver would necessarily be derived from transitions in the received data-signal and phase-jitter caused by noise and interference would perturb the instant of resampling. However, the main object of the experiment was to determine the fundamental differences between the three codes and clock-synchronisation was considered to be a separate (albeit important) issue. For this reason, a 'clean' feed of line-locked clock pulses employed in forming the data sequence was used to re-sample the distorted data signals. Separate measurements in the field have shown that, even with NRZ-coded signals as used in CEEFAX, clock synchronisation is not a serious problem; further details are given later in Section 4.3.

The slicing level was adjusted to give a 1 : 1 mark-to-space ratio in the sliced output signal, with a 10101010. . . data-sequence input to the slicing circuit, under no-noise conditions.

After translation to convenient (TTL) logic levels, the recovered data sequence was, as already outlined, decoded into NRZ code (if necessary) and fed into a comparator where it was compared with the 'reference' data sequence. The reference sequence had, necessarily, to be delayed so that corresponding data bits in the recovered and reference sequences arrived at the comparator in synchronism. Whenever a signal data bit was incorrectly decoded due to noise or interference of another kind, the comparator gave an output pulse; these pulses were counted to give the absolute bit error-rate in bit-errors per-second. Further, by inverting the reference data before comparison, and using a noise-free undistorted data signal, one error-pulse per bit could be generated, thus indicating the actual bit-rate in bits-per-second. The ratio of these two counts then was used to obtain the relative error-rate in bit-errors per bit received. The signal-to-noise ratio was adjusted in each measurement to give a mean bit-error rate of 1 in 10^5 averaged over 25 samples; this bit-error rate was chosen as a standard for comparison representing acceptable, if imperfect, reception.

3.3. Comparisons

Using the method set out in 3.1, two comparisons were made. First NRZ at 4·5 Mbits/sec was compared with BiφL at 2·25 Mbits/sec; these bit-rates were chosen because, in principle, BiφL provides about half the bit rate within the same bandwidth. Second 4·5 Mbits/sec NRZ was compared with 4·5 Mbits/sec D.M.

4. Results

4.1. NRZ and BiφL compared

Table 1 shows the signal-to-noise ratios for 4·5 Mbits/sec NRZ and 2·25 Mbits/sec BiφL for bit-error rates of 1 in 10^5 against the base bandwidth imposed on the data signals-plus-noise.

From these results can be deduced that NRZ at a given bit-rate behaves in much the same way as BiφL at half the bit-rate. Further, it appears that pseudo-random raised-cosine NRZ-coded data can be bandlimited (at least to 0·6 bit-rate) without significantly altering its resistance to noise. Although the modifications to the spectrum discussed in Section 2.5 have not been considered here, the ability of NRZ-coding to withstand bandlimiting is obviously important because it allows the possibility of increasing the bit-rate to a frequency beyond the base bandwidth of a video signal.

Table 1 also includes the result of measurement on NRZ with a 1·6 MHz filter. In this case the bandwidth was below the Nyquist limit of 2·25 MHz and very high error-rates resulted even from very low noise conditions.

4.2. NRZ and Delay Modulation compared

Table 2 shows the signal-to-noise ratios measured for 4·5 Mbits/sec NRZ coding and for 4·5 Mbits/sec D.M. coding for the same error rate of 1 in 10^5, for two base bandwidths.

Comparing Table 2 with Table 1, it will be seen that there is a discrepancy (about 1 dB) in the signal-to-noise ratio required for 1 in 10^5 error rate at a bandwidth of 3·4 MHz for NRZ (Table 2) as compared with an inter-

TABLE 2

NRZ Compared with D.M.

Bandwidth (−3 dB) MHz	Bandwidth / Bit-rate	NRZ (4·5 Mb/sec) Signal-to-noise ratio for 1 in 10^5 error rate	D.M. (4·5 Mb/sec) Signal-to-noise ratio for 1 in 10^5 error rate
4·5	1·00	19·8 dB	21·2 dB
3·4	0·76	20·7 dB	27·4 dB

polated value from Table 1 (this is due to experimental error). However, the results in Table 2 show that there is a significant difference, between NRZ and D.M., when using a 3·4 MHz bandwidth which indicates the serious effect of bandwidth-limiting a D.M. signal.

A particular disadvantage of D.M. code was found to be the criticality of the timing of the decoding clock pulses. Because D.M.-coded data requires sampling twice per bit-cell unambiguously to decode each bit, 9 MHz clock-pulses are required to decode 4·5 MHz bit-rate D.M. When intersymbol interference is present on the data signal (for example as a result of using a sharp-cut low-pass filter) the position of clock-pulse edges is particularly critical for the correct decoding of D.M.

For the same data rate, D.M.-coded data will therefore be more susceptible than NRZ-coded data to group-delay distortion in the transmission path. Group delay distortion invariably results in intersymbol interference.

4.3. Clock synchronising

Field measurements using a mobile laboratory have been made on the phase-jitter of clock-pulses derived from transmitted CEEFAX data signals. These clock-pulses were regenerated from data transitions in the signal using a simple, high-Q tuned circuit such as might be used in a data-broadcasting receiver.

The measurements showed only about 15° peak-to-peak phase jitter in the clock-pulses thus regenerated, under reception conditions bad enough to result in an effective data bit-error rate of 1 in 10^2. The experiments described in Section 3 involve considerably lower bit-error rates than these. The use of clean clock-pulses for resampling of processed data does not therefore invalidate the application of the results of Section 4.1 and 4.2 to the reception of broadcast data.

4.4. Multipath effects

A multiple transmission path will result in intersymbol interference in the demodulated data-signal. In the simplest case of a single low-level echo, the intersymbol interference comprises a delayed and attenuated version of the main signal added to that signal. When decoded the distorted data signal will probably be more susceptible to noise; the noise-immunity to errors of the code will have been reduced. Multipath effects on NRZ coded signals have not been considered in this Report; the choice of secondary-code for CEEFAX was made on the other criteria mentioned in previous sections.

Furthermore group-delay distortion anywhere in the signal chain (transmitter and/or receiver) gives rise to intersymbol interference and makes the data signal more susceptible to noise. This study has assumed the group-delay distortion to be low enough to be ignored.

5. Conclusions

For secondary-coding in a practical data-broadcasting system such as CEEFAX Non-Return-to-Zero code offers distinct advantages over BiPhase Level code and Delay Modulation code when transmitted as raised-cosine pulses.

Within a given bandwidth, NRZ can be transmitted at at least twice the bit-rate of BiϕL for the same error resistance to noise.

NRZ also withstands bandwidth limiting more readily than Delay Modulation, primarily because of the effect of intersymbol interference on Delay Modulation.

Where larger bit-rates are employed, and the band limit is closer, the magnitude of the impairments will change. However subsequent results suggest that the relative value of the impairments remain unchanged and, for example, the conclusion drawn from this work is equally applicable to bit-rates of the order of 6·9 Mbits/sec.

6. References

1. HECHT, M. *and* GUIDA, A. 1969. Delay-Modulation *Proc. IEEE*, July 1969, pp. 1314 – 1316.

2. INGLETON, J.G. A low-pass filter for television applications. BBC Research Department Report No. 1963/35.

Appendix A

Power Spectral Densities of Raised-Cosine Coded Data Pulses

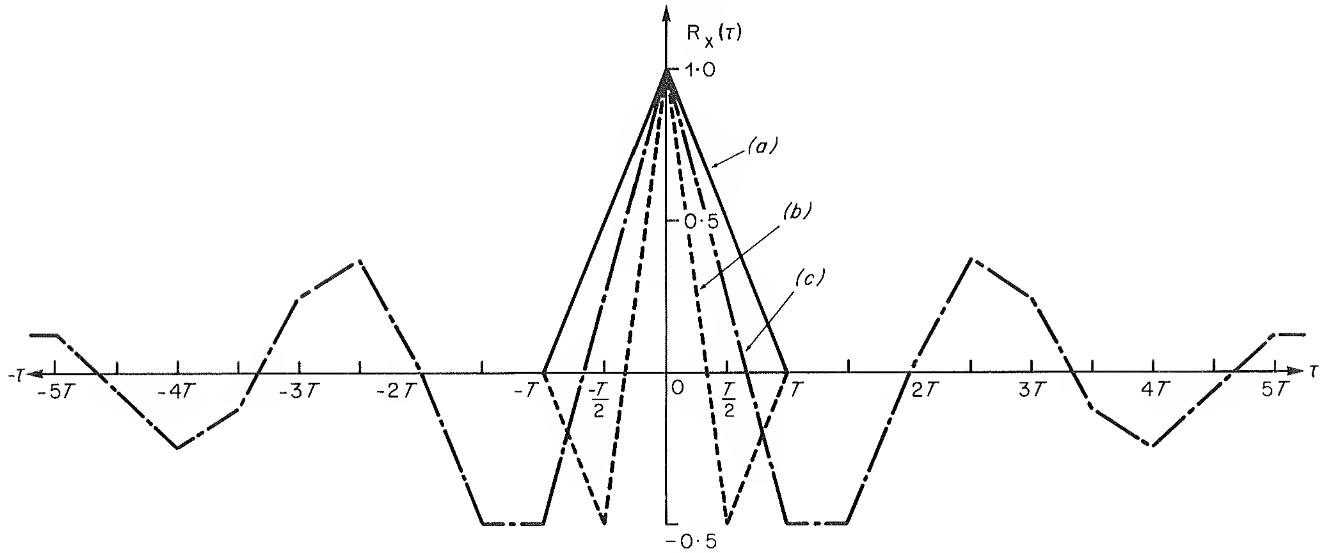

Fig. A1 - Autocorrelation functions of NRZ, BiφL and D.M. codes, vertical scale in normalised units $A^2 = 1$

(*a*) ———— NRZ (*b*) - - - - - - - BiφL (*c*) ——– – —— D.M.

The single-sided power spectral densities of the three codes considered are calculated from the autocorrelation functions of the coding methods (see Fig. A1, and Reference 2) and from the raised-cosine filter responses of the appropriate filters (see Appendix B).

The single-sided power spectral densities are thus as follows:

(i) For NRZ Code

$$S_i(\omega) = \frac{2A^2 T \sin^2(\omega T)}{(\omega T)^2 [1 - (\omega T/\pi)^2]^2}$$

(ii) For BiφL Code

$$S_{ii}(\omega) = \frac{2A^2 T . \sin^2(\omega T/2) . \sin^2(\omega T/4)}{(\omega T/2)^2 \left\{1 - (\omega T/2\pi)^2\right\}^2}$$

(iii) For D.M. Code

$$S_{iii}(\omega) = \frac{A^2 T \cos^2(\omega T/4) . \sum}{(\omega T/2)^2 (1 - (\omega T/2\pi)^2)^2 \left\{17 + 8\cos(\omega T/2)\right\}}$$

where $$\sum = 23 - 2\cos(\omega T/2) - 22\cos(\omega T) - 12\cos(3\omega T/2) + 5\cos(2\omega T) + 12\cos(5\omega T/2)$$

$$- 3\cos(3\omega T) - 8\cos(7\omega T/2) + 2\cos(4\omega T)$$

where ω angular frequency, T bit-cell clocking interval and A peak-to-peak signal amplitude.

Appendix B

Raised-Cosine Filter

The required filter transfer function in the S plane is

$$G(s) = \frac{1 + \exp(-st)}{2\left\{1 + (st/\pi)^2\right\}}$$

This was approximated with a fourth-order Gaussian transitional filter of the form

$$G^*(s) = (C_0 s^4 + C_1 s^3 + C_2 s^2 + C_3 s + C_4)^{-1}$$

which can be written as

$$G^*(s) = \frac{(\sigma_1{}^2 + \omega_1{}^2).(\sigma_2{}^2 + \omega_2{}^2)}{(s^2 - 2\sigma_1 s + \sigma_1{}^2 + \omega_1{}^2).(s^2 - 2\sigma_2 s + \sigma_2{}^2 + \omega_2{}^2)}$$

where $(\sigma_i, \pm\,\omega_i)$ are the two pole-pairs.

This factorises as

$$G^*(s) = \frac{A_1 s + B_1}{(s^2 - 2\sigma_1 s + \sigma_1{}^2 + \omega_1{}^2)} + \frac{A_2 s + B_2}{(s^2 - 2\sigma_2 s + \sigma_2{}^2 + \omega_2{}^2)}$$

In the time domain, the step response of the filter is

$$y(t) = u(t)\left\{\begin{aligned} &1 + \exp(\sigma_1 t)\left\{\frac{-B_1\cos(\omega_1 t)}{(\sigma_1{}^2 + \omega_1{}^2)} + \frac{1}{\omega_1}\cdot\left(A_1 + \frac{\sigma_1 B_1}{(\sigma_1{}^2 + \omega_1{}^2)}\right).\sin(\omega_1 t)\right\} \\ &+ \exp(\sigma_2 t)\left\{\frac{-B_2\cos(\omega_2 t)}{(\sigma_2{}^2 + \omega_2{}^2)} + \frac{1}{\omega_2}\cdot\left(A_2 + \frac{\sigma_2 B_2}{(\sigma_2{}^2 + \omega_2{}^2)}\right).\sin(\omega_2 t)\right\}\end{aligned}\right\}$$

The response of the filter to a rectangular pulse of duration τ is

$$y'(t) = y(t) - y(t - \tau)$$

The coefficients σ_i, ω_i are chosen so that the time between the mid-height points of $y'(t)$ equals τ and so that $y'(t)$ reaches a maximum half way in time between the mid-height points of $y'(t)$. Fig. B1 shows the form of filter used, the component values having been chosen to satisfy the criteria for σ_i, ω_i just mentioned.

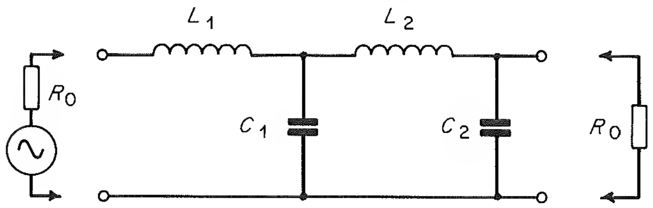

Fig. B1 - Fourth-order Gaussian transitional filter used for raised-cosine filtering of rectangular data pulses

R_0 characteristic impedance (Ohms)
$L_1 = 0{\cdot}10855\,R_0/T$; $L_2 = 0{\cdot}29989\,R_0/T$
$C_1 = 0{\cdot}20251\,T/R_0$; $C_2 = 0{\cdot}55416\,T/R_0$
where T duration of clocking interval (seconds), L in Henries, C in Farads

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